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## Elliptic Fourier analysis of facial profiles during growth and development

*The quantitative analysis of facial soft tissues is of overwhelming importance for orthodontic patients. To assess the normal age-related variations in shape, soft tissue facial profiles were studied in 96 healthy male children 3 to 11 years of age and 16 young men (age 18). Standardized left-side photographs were taken of each subject, and facial profiles were traced between trichion and cervical point. The line joining the 2 landmarks was set as the baseline, and each outline was automatically digitized and mathematically reconstructed by a 30-harmonic elliptic Fourier series expansion. The same soft tissue profile was traced and digitized from the Bolton standards of comparable age. All profiles were standardized to the same area, and shape modifications were quantified by calculating the morphologic distance between the Fourier reconstructions of each facial profile and of (1) the 18-year-old Bolton standard (MD-18) and (2) the age-related Bolton standard. Descriptive statistics were computed for each age class. On average, MD-18 was 7.23 at 3 years of age, increased between 6 and 9 years of age, and decreased hereafter, reaching 6.86 at 18 years of age. Within-group variability peaked at 8 years of age, and was minimal at 6 and 18 years of age. The current soft tissue child profiles seemed different from the profile obtained from the Bolton standards. (Int J Adult Orthod Orthognath Surg 2002;17:348–354)*

The quantitative analysis of facial soft tissues is an essential part of the diagnosis of dentofacial alterations and deformities. Although the evaluation of facial morphology can presently be performed in all 3 spatial dimensions, and technology supplies several instruments for the quantitative analysis of soft tissues,<sup>1–9</sup> in orthodontics, facial profiles are still evaluated on 2-dimensional reproductions (radiographs and photographs).<sup>1,10–20</sup> Indeed, not only classic reference standards were mainly developed for separate 2-dimensional views,<sup>21</sup> but in almost all orthodontic practices only 2-dimensional instruments can be found, and the use of 3-dimensional digitizers is limited to research laboratories.<sup>2–9</sup>

During growth and development, all craniofacial structures undergo modifications in their size, shape, and reciprocal arrangement.<sup>1–4,6,10–12,21,22</sup> Size modifica-

tions are of larger magnitude and can mask shape variations.<sup>1,7,17,18,22</sup> While Euclidean measurements (distances, areas, volumes) sufficiently evaluate the dimensions (size) of craniofacial components, facial profile is a complex factor that cannot be reduced to Euclidean geometry and accurately described by conventional metric measurements.<sup>13,17,18,20,23,24</sup> More precise analyses of the shape of biologic structures can be performed by Fourier series. This method mathematically describes the outline of objects and can quantitatively analyze their global shape characteristics, independent of their size, spatial orientation, or relationship to reference planes.<sup>25</sup> Both classic Fourier series and elliptic Fourier analysis have already been successfully applied to the quantitative study of biologic forms in dentistry, and recent extensive reviews can be found elsewhere.<sup>19,26</sup>

Table 1		Fourier analysis of the soft tissue profiles of 112 healthy males			
Age (y)	n	MD-18	CV-18 (%)	CV-age (%)	Bolton vs 18
3	16	7.23	21.30	26.70	8.01
6	16	7.25	16.41	21.01	5.15
8	16	7.29	26.61	26.31	4.81
9	16	7.38	18.70	24.94	6.43
10	16	7.28	20.60	27.55	4.60
11	16	7.18	18.52	18.63	3.32
18	16	6.86	16.03	16.03	0.00

MD-18 = mean value of the morphologic distance between each size-standardized profile and the 18-year-old Bolton standard profile (absolute number); CV-18 = coefficient of variation of the morphologic distance between each size-standardized profile and the 18-year-old Bolton standard profile (percentage); CV-age = coefficient of variation of the morphologic distance between each size-standardized profile and the age-related Bolton standard profile (percentage); Bolton vs 18 = morphologic distance between the size-standardized Bolton standard profile of each age and the 18-year-old Bolton standard profile (absolute number).

In the present study, the age-related modifications in the shape (size-independent) of facial soft tissue profiles were analyzed in a cross-sectional group of male children by means of elliptic Fourier analysis. Data were compared to the reference outlines obtained from the Bolton standards of dentofacial developmental growth, which represent a norm for comparative qualitative and quantitative measurements.<sup>21</sup>

## Materials and methods

### Sample

Ninety-six male children (3 to 11 years of age) and 16 young men (18 years of age) entered the present study (Table 1). The subjects, all healthy white Caucasians (Northern Italians), were randomly selected from the database of the Laboratorio di Anatomia Funzionale dell'Apparato Stomatognatico (University of Milan, Italy). Subjects with a history of craniofacial trauma, congenital anomalies, surgery, or orthodontics were not included in the sample. Data on these subjects were collected during longitudinal and cross-sectional growth studies that are currently being performed at Milan University.<sup>2-4,6,7,13</sup> Each subject entered the present study once only.

All assessments were noninvasive and were performed with procedures currently not known to involve any present or future biologic damage. All subjects and the parents of children younger than 18 years of age were informed about all the adopted procedures and gave their consent to the investigation. The study protocol was approved by the local ethics committee.

### Data collection

For all subjects, the standardized procedure described by Ferrario et al<sup>13,20</sup> was followed. In brief, left-side photographs were taken while the subjects stood 2.5 m away from the camera and looked straight ahead. Each subject was asked to assume and maintain a natural head and body position. The camera was mounted on a tripod and leveled, with the optical axis of the lens horizontal and the film plane vertical. The films were developed and printed with a final magnification fixed at  $\times 0.62$ .

To reduce measurement error, all facial profiles were traced and digitized by a single operator as detailed by Ferrario et al.<sup>20</sup> Facial profiles were traced between 2 standardized landmarks: trichion<sup>27</sup> and cervical point (the junction between the neck and the throat).<sup>15,16</sup> The line joining the 2 landmarks was set as the baseline (Fig 1).



**Fig 1** Left-side photograph of a child aged 3. Trichion (Tr) and cervical point (Cp) were used as endpoints for digitization of the profile.

The soft tissue facial profiles of the annual Bolton standards<sup>21</sup> for ages 3, 6, 8, 9, 10, 11, and 18 (male and female average) were also traced using the same standardized procedure.

All tracings were then digitized using a scanner interfaced to a personal computer, and the facial outline was obtained by means of a computerized program (Morphological Analyzer; CUBE, Milano, Italy).<sup>26,28</sup> For each outline, the x and y coordinates of approximately 250 points were obtained, with the y-axis coinciding with baseline.

#### *Fourier analysis of facial profiles*

For each facial profile, the x,y coordinates of the points were encoded with a numeric code (or chain code), which provided a description of the profile, and allowed the calculation of the relevant elliptic Fourier coefficients.<sup>23,29</sup>

The elliptic Fourier series expansions for the x and y projections of the contour are defined as follows:

$$x(t) = A_0 + \sum_{n=1}^{30} \left( a_n \cos \frac{2n\pi t}{T} + b_n \sin \frac{2n\pi t}{T} \right)$$

$$y(t) = C_0 + \sum_{n=1}^{30} \left( c_n \cos \frac{2n\pi t}{T} + d_n \sin \frac{2n\pi t}{T} \right)$$

where  $A_0$  and  $C_0$  are the coordinates of the harmonic centroid;  $a_n$ ,  $b_n$ ,  $c_n$ , and  $d_n$  are the 4 coefficients of the nth elliptic harmonic; and  $0 < t \leq T$ , where T is the basic period of the chain code.<sup>29</sup>

The elliptic Fourier series were normalized with respect to the rotation, translation, and size of the contour. Standardization for rotation was performed by aligning the semimajor axis of the first harmonic on the abscissa; for translation, by ignoring the  $A_0$  and  $C_0$  terms; and for size, by dividing each coefficient by the magnitude of the first semimajor axis.<sup>29</sup> Fourier series were truncated at the 30th harmonic, because the higher-degree coefficients and relevant amplitudes were negligible (Fig 2).

A "morphologic distance" (MD), ie, a measurement of differences in shape between each facial profile and the 18-year-old Bolton standard profile, was computed using the Fourier coefficients calculated for each outline (all four a, b, c, d coefficients of the first 30 elliptic harmonics).<sup>20,23,26,28,29</sup>

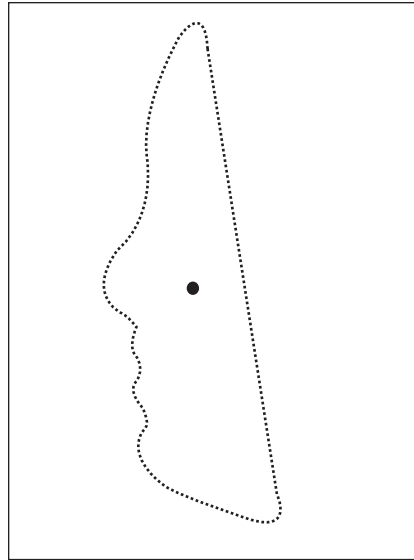
The same calculation was also performed between each profile and his age-related Bolton standard profile, and a second series of MD values was obtained. Furthermore, each age-related Bolton standard profile was compared to the 18-year-old Bolton standard profile.

The MD measures the Euclidean distance in a 30-dimensional space (the first 30 harmonics) between 2 plots characterized by their Fourier coefficients, which are used similar to Cartesian coordinates in standard metric measurements. The MD equals 0 when the profiles are identical, and when they have been sampled with the same lattice coarseness and orientation.<sup>20,23,26,28,29</sup> For easier reading, all MDs were multiplied by 100. The MD quantifies the difference between 2 outlines with a single, comprehensive number that incorporates all the morphologic characteristics of the analyzed structures.

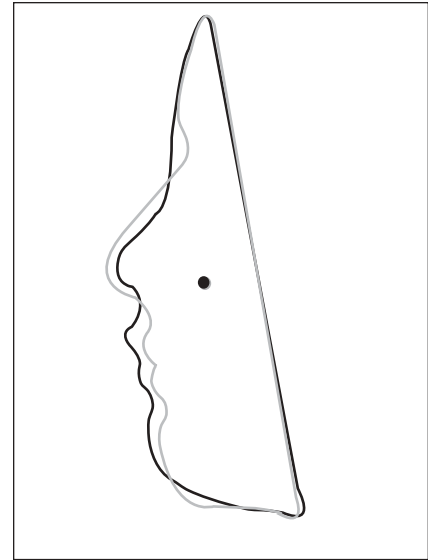
#### *Statistical calculations*

Descriptive statistics (mean, standard deviation, coefficient of variation, percentage ratio of standard deviation to mean) of the morphologic distances calculated versus (1)

**Fig 2** (left) Elliptic Fourier reconstruction (30-harmonic truncation) of the facial profile of the child shown in Fig 1. For illustration purposes, the profile was oriented in natural head position.



**Fig 3** (right) Elliptic Fourier reconstruction (30-harmonic truncation) of the facial profile of the 18-year-old Bolton standard (gray line) as compared to the facial profile of a 3-year-old child (black line). The profiles have been superimposed on the center of gravity (dot) and on the axes of the first elliptic harmonic. The MD is 5.61.



the 18-year-old Bolton standard (MD-18) and (2) the age-related Bolton standard (MD-age) were computed separately for each age class.

## Results

All the soft tissue facial profiles used for the present study were reconstructed well by elliptic Fourier series, and in all cases, the 30-harmonic truncation produced outlines that were easily superimposed on the original tracings (Fig 2).

To obtain a common reference, each child profile was compared to the 18-year-old Bolton standard (Table 1, Fig 3). Overall, a slight increase of the mean value of the distance MD-18 was observed between 6 and 9 years of age, with a progressive decrease thereafter. A similar behavior was found for the MD between each age-related Bolton standard profile and the 18-year-old profile: a decrease in the mean values these between 3 and 8 years of age, a somewhat larger difference at 9 years of age, and progressively smaller differences thereafter. Indeed, apart from the comparison at 3 years of age, at all ages the 18-year-old Bolton profile was more similar to the age-related Bolton standards than to the current children (smaller MDs).

Within-group variability in the current sample, as assessed by the coefficient of variation, peaked at 8 years of age and was minimal at 6 and 18 years of age (Table 1).

To better assess the age-related variability, each child profile was also compared to the age-related Bolton standard, and the relevant coefficient of variation (CV-age) was computed. The lowest values (ie, the least within-sample variability) were found at 6 and 11 years of age.

## Discussion

One of the main components of the clinical analysis of patients with facial alterations and deformities is the quantitative assessment of the dimensions, reciprocal spatial positions, and relative proportions of the facial soft tissue structures (eyes, nose, lips, chin, ears).<sup>11-18,20</sup> Treatment planning should also consider how skeletal movements will affect the soft tissue appearance, because the final evaluation of results is made by the patient and her/his mirror.<sup>15,16</sup>

The quantitative analysis of facial profile is complex, and a large number of angular and linear measurements, as well as ratios, have been proposed for its evaluation.<sup>10-16</sup> Unfortunately, they often cannot separate the contributions of size and shape to the global morphology.<sup>13,17,19,20,24</sup> If only the profile outline is considered, the measurements can be performed by using a mathematic analysis, namely Fourier series.<sup>13,19,20,23-26,29</sup>

This method mathematically reconstructs an object outline by a sum of sine and cosine functions of increasing frequency, and it can separate the size and

shape components.<sup>13,19,20,23–26,29</sup> The outline (facial profiles in the present study) is therefore described by a number of coefficients, which can be used for statistical comparisons.<sup>24</sup> Two different methods have been used for Fourier reconstruction of biologic shapes: classic Fourier methods,<sup>24</sup> in which each harmonic is defined by 2 coefficients, and elliptic Fourier methods,<sup>29</sup> with 4 coefficients for each harmonic. Currently, elliptic Fourier analysis is considered more suitable for the analysis of biologic forms, provided that a closed contour exists. A detailed discussion can be found elsewhere.<sup>19,25</sup> In brief, elliptic series allow an internal (self-consistent) orientation of structures for comparison: each form is rotated until the major axes of the first harmonic ellipses coincide.<sup>29</sup> Such an internal orientation cannot be performed by classic Fourier analysis.

Moreover, the 4 coefficients of each elliptic harmonic can be used similar to Cartesian coordinates of standard metric measurements, and a mathematic “distance” can be computed between pairs of outlines.<sup>20,23,26,28,29</sup> A single number, namely the “morphologic distance” calculated in the present investigation, can thus differentiate between 2 structures.

Therefore, the child profiles analyzed in the current study were reconstructed by elliptic Fourier series. The outline was closed by tracing a baseline from trichion to cervical point (Fig 1), a procedure followed in other investigations.<sup>20</sup> In contrast, Tangchaitrong et al<sup>19</sup> closed the soft tissue facial profiles of their children by reflecting the profile around a vertical reference axis, thus creating a symmetric outline. The subsequent mathematic reconstruction was similar to that used in the present study: the profiles were sampled using approximately 200 points, and the elliptic Fourier analysis was performed with a 30-harmonic truncation. In contrast, the comparisons were made on a single harmonic and coefficient basis.<sup>19</sup>

One of the major findings of the present study is the limited difference in the shape of the soft tissue profile between 3 and 18 years of age (Fig 3). At 3 years of age, the average difference between the child soft tissue profile of the current sample and the reference “adult” profile (the Bolton stan-

dard at age 18, the oldest reference, which should represent the attainment of “maturity”)<sup>21</sup> was only 5% greater than that measured between the 18-year-old men and the same Bolton standard (Table 1). In the analyzed period of postnatal growth, the MDs showed that, on average, the size-standardized difference between the current child profile and the 18-year-old Bolton standard increased slightly between 3 and 9 years of age (about 2%) and then decreased in the oldest age groups (about 8%). When each age-related Bolton standard profile and the 18-year-old Bolton profile were compared, the differences increased, ranging from 8.01 at 3 years of age to 3.32 at 11 years of age (Table 1). Apart from the 3-year-old-profile, at all ages the comparison between the 18-year-old Bolton profile and the age-related Bolton standard yielded MDs that were 0.46 (11 years of age) to 0.87 (9 years of age) times smaller than that obtained comparing the current children to the same reference.

These findings seem to indicate that the current soft tissue child profiles are different from those obtained from the Bolton-Brush growth study.<sup>21</sup> Those classic standards might now be inadequate for the quantitative analysis of normal children. Indeed, several factors differ between the present children and those used for the development of the Bolton standards: some of the Bolton children were born in the 1930s, while the oldest of the present subjects were born in the 1970s. The Bolton children were all Northern Americans with Northern European ancestry; in contrast, the present children and young men were all Northern Italians, probably more similar to a “Latin” ethnic group.<sup>9</sup> A secular trend as well as an effect of ethnicity cannot be excluded.<sup>30</sup>

It must be emphasized that the MD compared only the shape (size-independent) of the facial outline. Overall, facial development is largely completed by age 5, and most facial growth that takes place after 5 years of age is dimensional.<sup>22</sup> Unfortunately, size-independent shape modifications in the facial profiles of children have been analyzed only rarely,<sup>19</sup> and comparisons with previous studies are difficult. The shape of the facial profile may be approximated by

other size-independent measurements. For instance, angular measurements can be considered a rough assessment of shape, even if they can be influenced more by local variations than a global measure such as the MD.

Indeed, during childhood and adolescence, several facial angles that can influence the soft tissue profile change very little or even not at all. For instance, the nasal prominence angle (subnasale-nasion-pronasale) measured by Ferrario et al<sup>2,6</sup> remained stable during this period, as did the maxillary prominence angle sublabiale-nasion-subnasale.<sup>6</sup> The angles of total facial convexity, both including the nose (nasion-pronasale-pogonion)<sup>6,31</sup> and excluding the nose (nasion-subnasale-pogonion)<sup>6,32</sup> were almost unchanged throughout the period considered, and no differences were observed between preadolescent and adult males.<sup>14</sup> Also, the angle glabella-subnasale-pogonion changed very little between 5 and 25 years of age.<sup>33,34</sup> The angular shapes and the positional relationships of the nose, lips, and chin remained relatively constant between 7 and 16 years of age,<sup>1,11</sup> and only minimal variations in the angles of soft tissue facial profiles were found.<sup>10</sup> Also, facial convexity in the horizontal plane did not change between 3 and 18 years of age, the largest change being a decrease of 3 degrees in the right tragion-pronasale-left tragion angle.<sup>3</sup>

Furthermore, in a 3-dimensional assessment of craniofacial growth, Ferrario et al<sup>7</sup> found that between 6 and 18 years of age most of the modifications in facial shape were concerned with pogonion and trichion. These 2 landmarks are positioned at the extreme limits of the present profiles, and, therefore, could have a limited influence on the shape of the soft tissue profile.

Together with the mean difference from the reference "adult" shape, within-group variability was also assessed, and the coefficients of variation of both MD-18 and MD-age (MD between each child profile and his age-related Bolton standard) were considered. It has to be mentioned that the mean value of MD-age is of little meaning, because the reference changes among the groups. The largest values for CV-age were found at 3 years of age and between 8 and

10 years, while CV-18 peaked at 8 years of age (Table 1). This finding may indicate a large variability in the individual growth patterns between 8 and 10 years of age, with children growing more or less rapidly.<sup>14</sup> Indeed, in a 3-dimensional study of nasal growth and development, Ferrario et al<sup>2</sup> found large variability in 8- to 9-year-old boys. Also, the interlabial angle ([subnasale-labrale superius]-[labrale inferius-pogonion]) was very variable (large standard deviation) between 7 and 11 years of age.<sup>6</sup> In contrast, limited variability in facial volume was found in boys aged 6.<sup>4</sup>

Among the limitations of the present study is the lack of some age groups. Indeed, data were collected during the longitudinal and cross-sectional growth studies that are currently being performed at Milan University,<sup>2-4,6-9,13</sup> but unfortunately, photographs were not taken for all children being examined. To obtain a balanced study, the same number of children for each group was used.<sup>17</sup> The smallest group (3-year-old boys) included 16 individuals, and the children from the other (more numerous) samples were randomly selected. Sixteen adult men were also added to provide a final reference for the comparison of the Bolton standards.

A further limitation pertains to sex. Only male subjects were examined because the study on the 3-year-old children did not include enough girls.<sup>8</sup> Therefore, all the present findings can be applied only to boys. Nevertheless, Tangchaitrong et al<sup>19</sup> reported that facial differences between sexes develop only after puberty.<sup>22</sup> Moreover, Ferrario et al<sup>7</sup> found small sex differences in the 3-dimensional facial shape between 6 and 11 years of age, and limited differences thereafter, which led them to state that sex-independent age-related norms may be used.

In conclusion, between 3 and 18 years of age a limited difference in the size-independent shape of the facial soft tissue profile was found, as compared to the Bolton standards.<sup>21</sup> This finding agrees with the negligible modifications observed during childhood and adolescence in several facial angles that can influence the soft tissue profile.<sup>1-3,6,10,11,14,31-34</sup> Moreover, a relatively larger variability was found between 8 and 10 years of age, with children growing at

different rates.<sup>14</sup> The differences were larger within the Bolton standard profiles,<sup>21</sup> a reference obtained from children who probably are not comparable to the present Italian children.

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